Fast Homomorphic Evaluation of Deep Discretized Neural Networks

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ENS, CNRS, PSL Research University, INRIA
(Work done while visiting CryptoExperts)

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Machine Learning as a Service (MLaaS)

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$\xrightarrow{\quad\quad x \quad\quad}$

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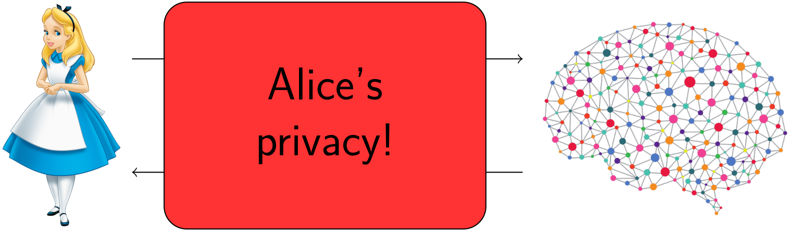

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- ✓ Privacy — data is encrypted (both input and output)
- ✗ Efficiency — main issue with FHE-based solutions

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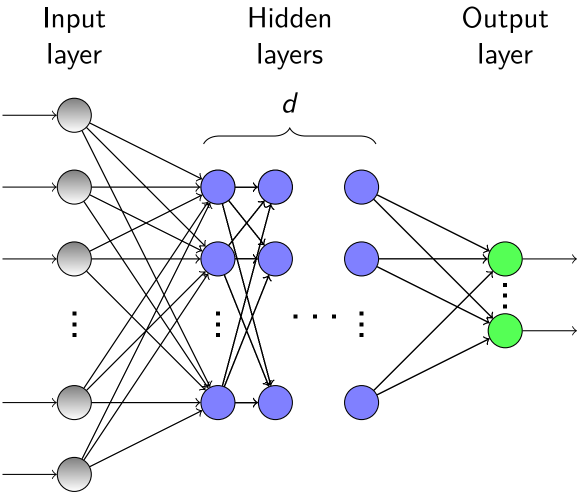
Goal of this work: homomorphic *evaluation* of trained networks.

(Very quick) refresher on neural networks



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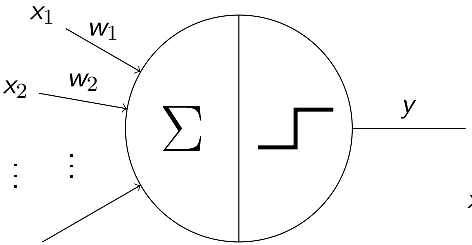
Computation for every neuron:



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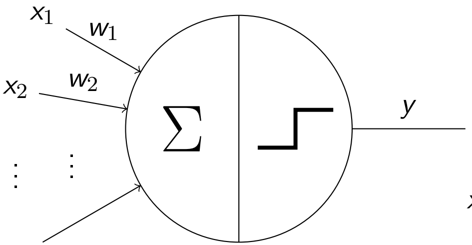

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$$y = f\left(\sum_i w_i x_i\right),$$

where f is an *activation function*.

A specific use case

We consider the problem of *digit recognition*.

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Dataset: MNIST ($60\,000$ training img + $10\,000$ test img).

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The computation at neuron level depends on the total multiplicative depth of the network $\implies$ bad for deep networks!

State of the art

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Goal: make the computation scale-invariant $\implies$ bootstrapping.

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We want to homomorphically compute the multisum

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Goal: *FHE-friendly* model of neural network.

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A DiNN is a neural network whose inputs are integer values in $\{-I, \dots, I\}$, and whose weights are integer values in $\{-W, \dots, W\}$, for some $I, W \in \mathbb{N}$.

For every activated neuron of the network, the activation function maps the multisum to integer values in $\{-I, \dots, I\}$.

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- Not as restrictive as it seems: e.g., binarized NNs;
- Trade-off between size and performance;
- (A basic) conversion is extremely easy.

Homomorphic evaluation of a DiNN

1. **Evaluate the multisum:** easy – just need a linearly hom. scheme

$$\sum_i w_i \cdot \mathrm{Enc}\left(x_i\right) = \mathrm{Enc}\left(\sum_i w_i x_i\right)$$

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Issues:

- **Choose the message space:** guess, statistics, or worst-case
- **The noise grows:** need to start from a very small noise
- **How do we apply the activation function homomorphically?**

Basic idea: activate during bootstrapping

Combine bootstrapping & activation function:

$$\mathsf{Enc}\left(x\right) \rightarrow \mathsf{Enc}^{*}\left(f\left(x\right)\right)$$

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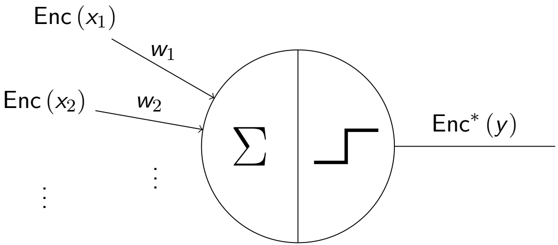

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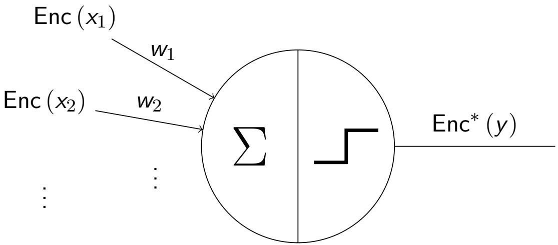

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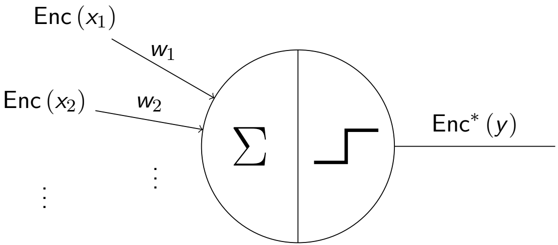

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Two steps:

1. Compute the multisum $\sum_i w_i x_i$
2. Bootstrap to the activated value

TFHE: a framework for faster bootstrapping [CGGI16,CGGI17]

$\mathbb{T} := \mathbb{R}/\mathbb{Z}$

Basic assumption: learning with errors (LWE) over the torus

$$(\mathbf{a},\ b = \langle \mathbf{s}, \mathbf{a} \rangle + e \mod 1) \overset{c}{\approx} (\mathbf{a},\ \mathbf{u}), \qquad e \leftarrow \chi_\alpha,\ \mathbf{s} \leftarrow_\$ \{0,1\}^n,\ \mathbf{a}, \mathbf{u} \leftarrow_\$ \mathbb{T}^n.$$

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Overview of the bootstrapping procedure:

1. Hom. compute $X^{b - \langle \mathbf{s}, \mathbf{a} \rangle}$: spin the wheel
2. Pick the ciphertext pointed to by the arrow
3. Switch back to the original key

Our activation function

We focus on $f(x) = \mathrm{sign}(x)$.

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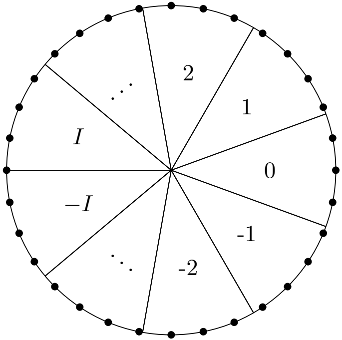

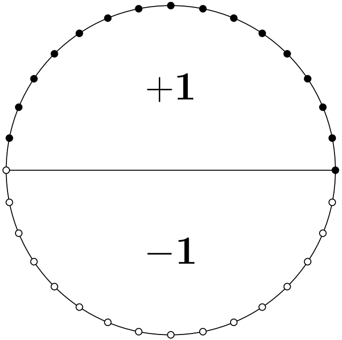

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The constant term of $ct \cdot w_{pol}$ is then $\text{Enc}\left(\sum_i w_i x_i\right)$.

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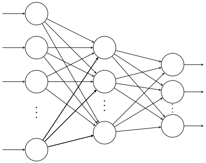
Bottom line

We can start with any message space at encryption time, and change it dynamically during the bootstrapping.

Overview of the process

Evaluation of a DiNN with 30 neurons in the hidden layer:



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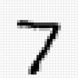
Server



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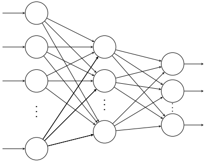

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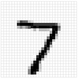

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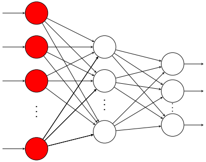

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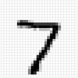

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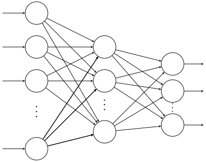

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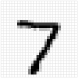

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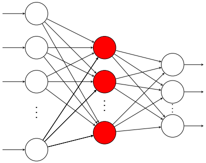

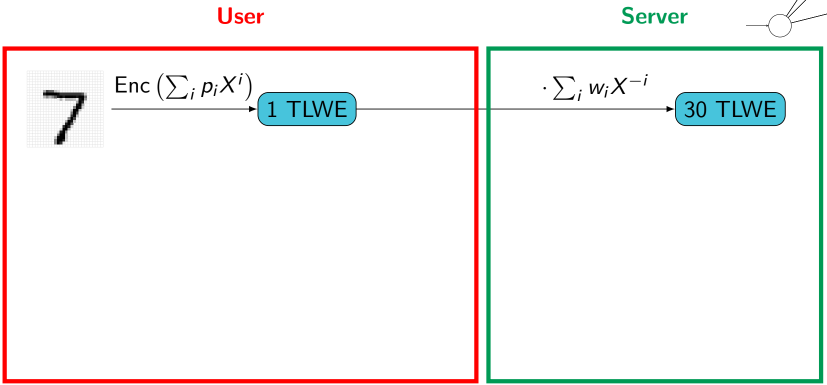

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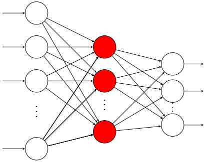

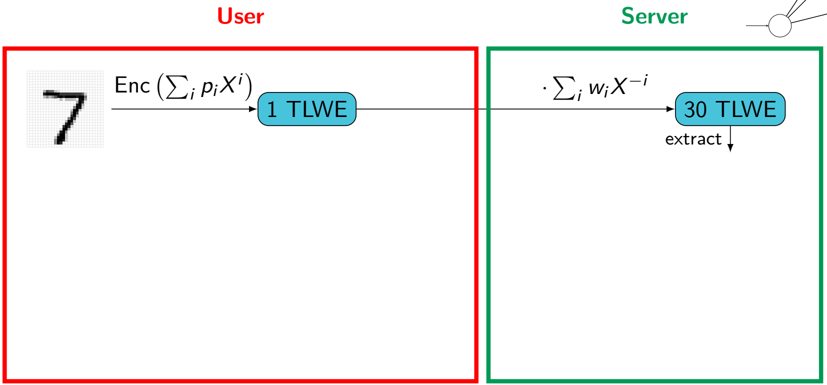

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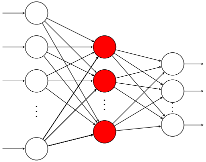

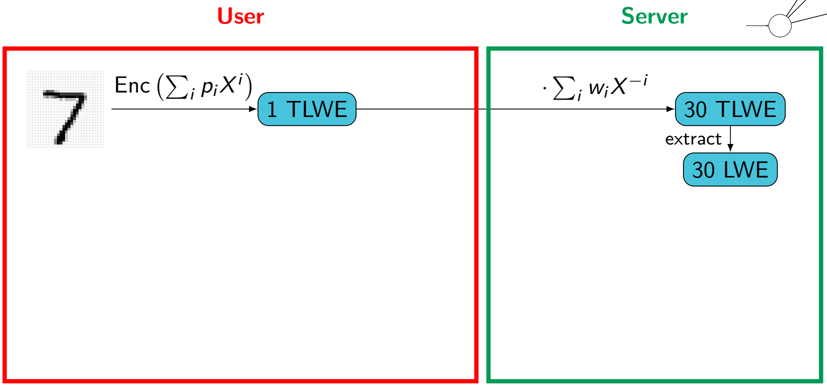

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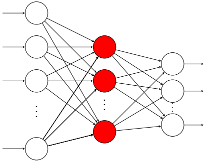

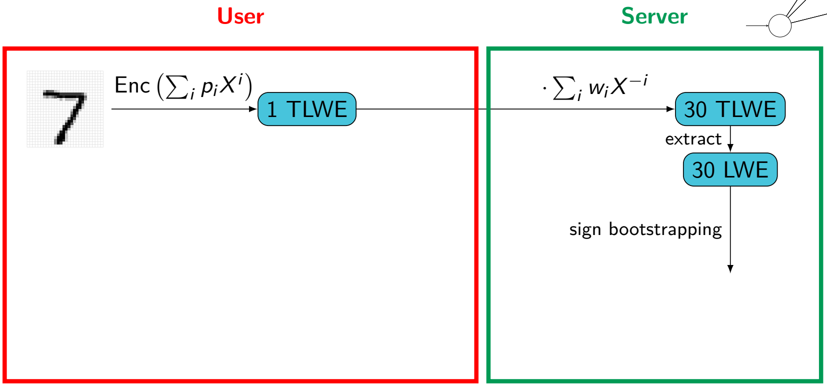

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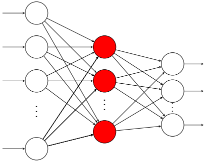

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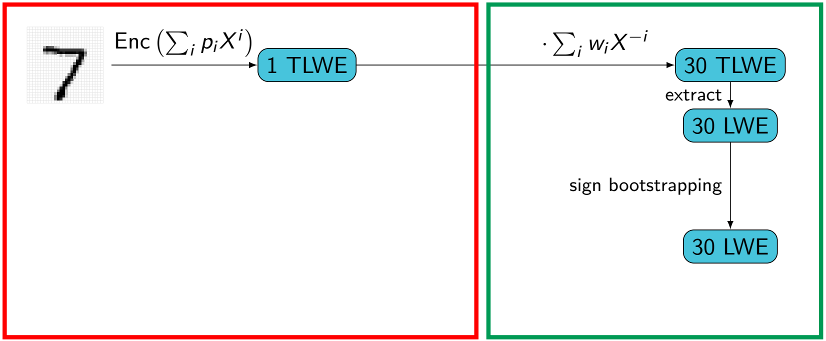

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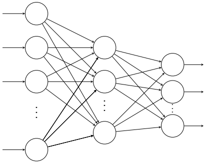

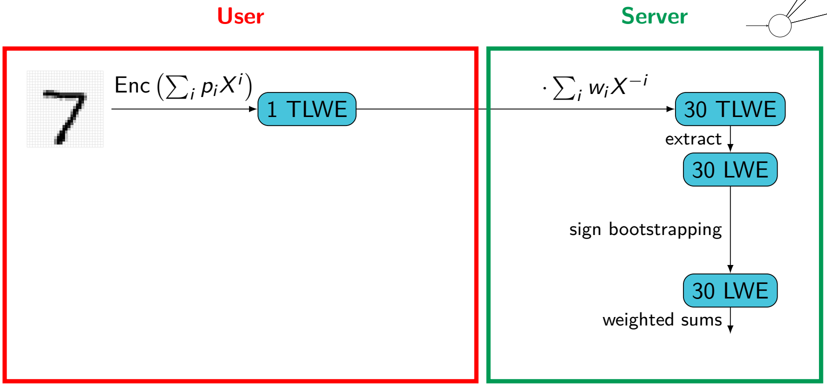

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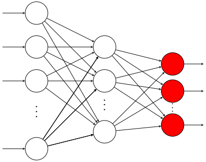

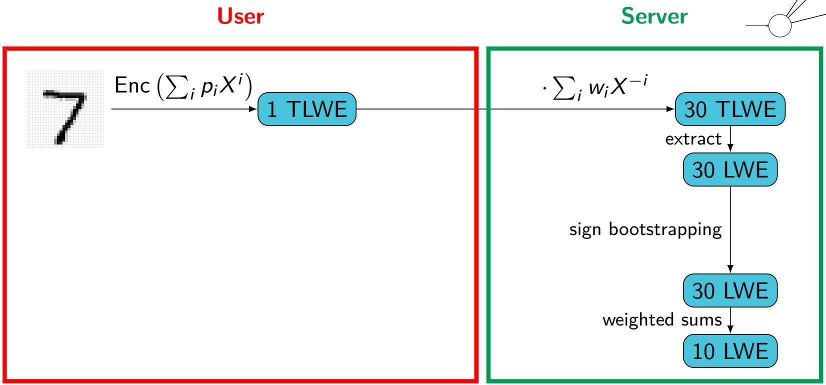

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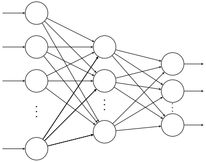

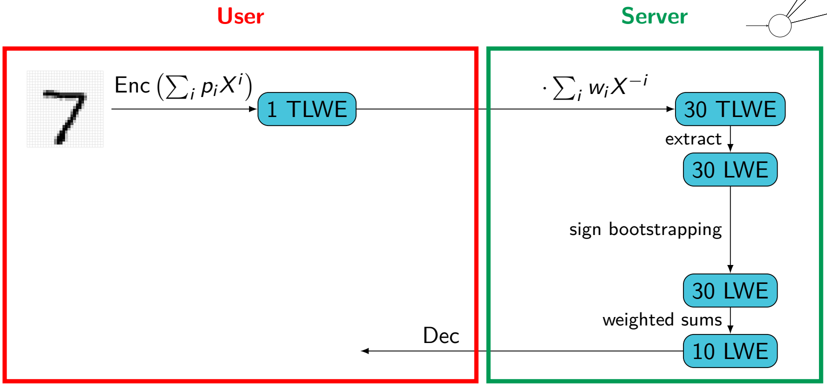

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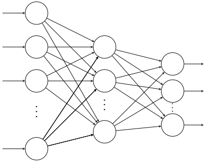

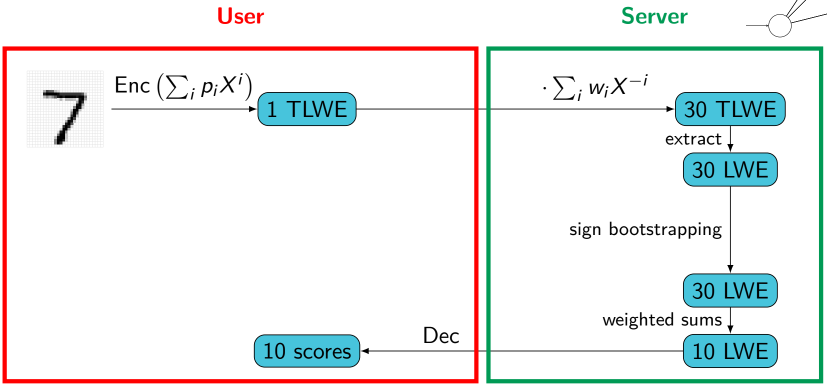

Overview of the process

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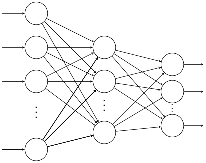

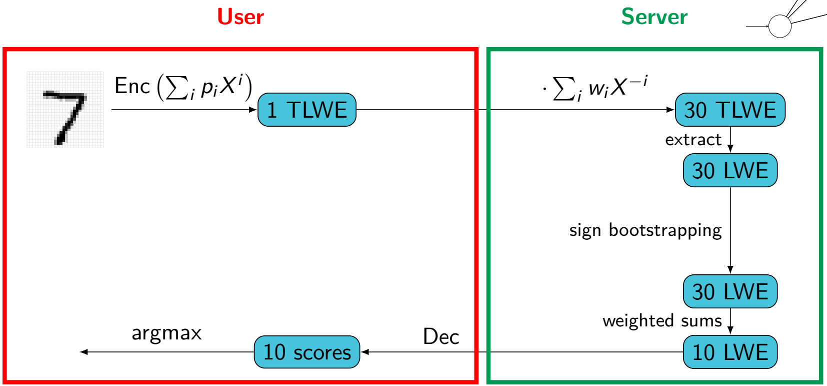

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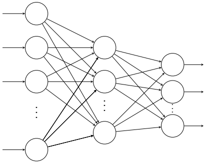

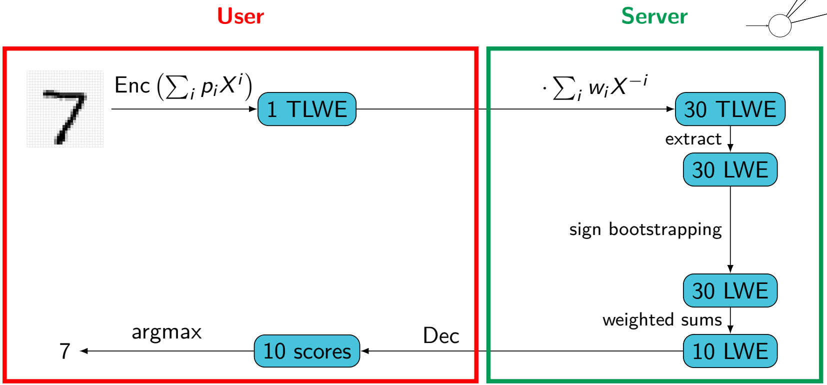

Experimental results

On inputs in the clear

	Original NN ($\mathbb{R}$)	DiNN + hard_sigmoid	DiNN + sign
30 neurons	94.76%	93.76% (-1%)	93.55% (-1.21%)
100 neurons	96.75%	96.62% (-0.13%)	96.43% (-0.32%)

On encrypted inputs

	Accur.	Disag.	Wrong BS	Disag. (wrong BS)	Time
30 or	93.71%	273 (105–121)	3383/300000	196/273	0.515 s
30 un	93.46%	270 (119–110)	2912/300000	164/270	0.491 s
100 or	96.26%	127 (61–44)	9088/1000000	105/127	1.679 s
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or = original   un = unfolded

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	Neurons	Size of ct.	Accuracy	Time enc	Time eval	Time dec
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independent of the network

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Thank you for your attention!

Questions?